# I. UNIFORM CIRCULAR MOTION

Uniform circular motion means an object that is going around in a circle at a constant speed. The velocity is continually changing as the direction of motion is always changing.

1. Centripetal Acceleration

Note that: (i) **the instantaneous velocity**, $\vec{v}$, is always tangential. This is the direction that a stone in a sling will actually go when the sling is released.
(ii) **the acceleration** is always towards the centre

Acceleration going towards the centre is called **centripetal acceleration, a_c.** There is no such thing as centrifugal acceleration – acceleration that pushes something to the outside of a circle. ♣Think about it a bit. When you are in a car that is turning to the left, you are <u>not</u> being pushed to the outside (the right). You want to keep on going straight (Newton's First Law). The reason that you don't go straight is that the car is moving to the left, the side of the car also pushes you **to the left**! – that is to the inside of the circle!
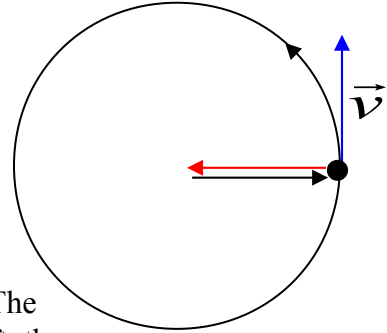
$\vec{v}$ and $\vec{a}$ are perpendicular. If this wasn't so, you wouldn't go in a circle. If $\vec{v}$ suddenly became zero, then the object would just go in the direction of $\vec{a}$ (the red arrow). If $\vec{v}$ got smaller slowly, you would spiral into the centre.

2. Formula

Any acceleration towards the centre is related to the velocity and radius:

$$a_c = v^2 / r$$

Obviously this is a <u>scalar</u> equation – it doesn't give direction as both $\vec{v}$ and $\vec{a}$ are constantly changing (so is $\vec{r}$ for that matter). *We also cannot divide vectors (we may be able to square them).*
Now for a circle, the distance around it is $2\pi r$ and the time for one revolution is T. $\therefore v = 2\pi r/T$ or $2\pi rf$.

This means that our formula for acceleration can be written as $a_c = 4\pi^2 r / T^2$ or $a_c = 4\pi^2 rf^2$

3. Centripetal Force

Any force that pushes something towards the centre of a circle is called **centripetal force**, F_c. This is the force that keeps something in circular motion. Centripetal Force is a "pseudo-force" in that there is always some other real force that is pulling the object in. Think of it as a way of describing or classifying other forces. Possible centripetal forces include gravity, tension, friction, the normal force, electromagnetic force.

All centripetal forces are also equal to $F_c = ma_c$ which gives $F_c = mv^2 / r$

<Nelson: p 126 #4 – 10>

NOT MUCH ABOUT GRAVITY – IT WILL ALL BE DONE IN THE FIELDS UNIT

II. CARS GOING AROUND CURVES

For problems involving cars going around a curve, we use centripetal force and another (real) force.
$F_c = mv^2/r$

The car will "want" to continue moving in a straight line, so there must be a force that pulls it in a circular path.
There are typically two forces that can do this on cars.

→ 1. Consider a flat, unbanked, curve. The force of friction (static friction since nothing is slipping) between the tires and the road is the centripetal force keeping the car on the circular road.
(How does this work? Think of a rollerblader – to go into a left turn he pushes his feet to the right: outward. The force of friction with the road causes the road to push him to the left.)
In this case, $F_f \geq mv^2/r$ Which way does the > < sign go? You can have an enormous force of friction and it won't affect things, but if it is too small, then the car will slide.
The meaning of mv^2/r is that this is the **minimum** force needed for a car to be going in a circle.

→ 2. Consider a banked curve. This is very nicely worked out in your new textbook.
If we ignore friction, then the force that is keeping the car in a circular path is the component of the normal force (from the road (perpendicular to the surface remember)) that points towards the centre of the curve.
You can calculate what this component is. (some function involving force of gravity and $\sin\theta$ or $\cos\theta$).
This component is then $\geq mv^2/r$.

Sample Problems:
→ 1. A flat unbanked curve. *Let's make up a problem.*
What are the variables? mass, co-efficient of friction, speed, radius of the curve.
Which three shall be given? Which one shall we solve for? r.
Let's pick some numbers: m = 9,000 kg (a transport truck?), μ = 1.5, v = 90 km/hr = 25 m/s, r = ? (I would guess a fairly wide curve – like 20 m).
I'll write this as a word problem: What is the smallest radius of curvature that a 9000 kg truck can navigate at 90 km/hr if the coefficient of friction between the tyres and the road is 1.5?

Ff = μ F$_N$ and F$_N$ = Fg   This frictional force is the centripetal force
  = 1.5 (9000kg)(9.8 N/kg) $\therefore$ $F_c = mv^2/r$
  = 132,300 N   Ff $= mv^2/r$
                                              $\mu mg = mv^2/r$
                                              solving for r: $r = v^2/\mu g$

   Hmm… by doing it algebraically, we see that the mass cancels out. You wouldn't notice this if you just started putting numbers in. Note that this problem does not consider the truck going so fast that it flips over. We are only considering sliding sideways off of the curve.

r = (25m/s)^2 / (1.5)(9.8 m/s^2)
  = 42.5 m

➔ 2. A banked curve with no friction (i.e. with a sheet of ice on it – like a bobsled run – if it is curved it can still keep the vehicle on the track.
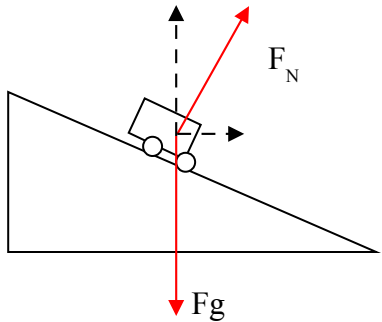*Let's make up a problem.*
What are the variables? mass, angle banked, speed, radius of the curve.
Which three shall be given? Which one shall we solve for? θ
Choose some numbers: m = 1000 kg, r = 15 m, v = 60 km/hr
Write a word problem with this: At what angle should a 30m diameter curve be banked for a 1000 kg car to be able to stay on the road at 60 km/hr assuming icy conditions?
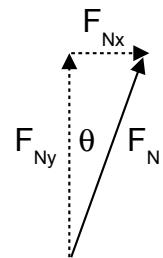
F_N

In this case it works best if we use x and y in the horizontal and vertical directions (because Fc is horizontal to the centre of the curve).
*** x is in the direction of the net force (Fc)

We can see two important things:
$F_c = F_{Nx}$ and $F_{Ny} = F_g$

Fg

F_{Nx}

F_{Ny} θ F_N

Draw a force triangle for F_N ➔
With some simple geometry, we can prove that the angle at the bottom of the triangle is θ - the same as the ramp angle.

From trig. we see that
$F_{Nx} = F_N \sin\theta$ and $F_{Ny} = F_N \cos\theta$ we won't need these relationship unless we need to find F_N

➔also: $\tan\theta = F_{Nx} / F_{Ny}$

If we substitute in from above … $\tan\theta = F_c / F_g$ $\tan\theta = (mv^2 /r) / mg$

.: $\tan\theta = v^2 / rg$ Note that we never had to calculate values for F_N or F_{Nx} ($F_{Ny} = F_g$)

Notice that the mass cancels out here too!
With numbers: $\tan\theta = (16.7m/s)^2/(9.8 N/kg)(15m)$
θ = 62.21 degrees.

In real life this angle would be less because friction also keeps the car on the road, and there would be speed reduction signs for such a tight curve.

Students (YOU) think that centripetal acceleration means that the speed is always increasing.
Constant speed in a circle = no acceleration!
But **acceleration** implies that **either** speed **or** direction changes (or both).